

Course

MATH 1C

Student ID

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Test

QUIZ 1

Question

2a

2b

Points

2 0

MAX: $6\frac{1}{2}$

MAX: $2\frac{1}{2}$

Question

3a

3b

3c

Points

7 $6\frac{1}{2}$ 6

MAX: 9

MAX: $9\frac{1}{2}$

MAX: $7\frac{1}{2}$

Total

$21\frac{1}{2}$

MAX: 35

$$\begin{aligned} [2][a] \quad a_{n+1} - a_n &= \frac{6^{n+1}}{(2n+3)!} - \frac{6^n}{(2n+1)!} \quad (1) \\ &= \frac{6^{n+1} - 6^n(2n+3)(2n+2)}{(2n+3)!} \quad (1) \\ &= \frac{6^n(6 - (2n+3)(2n+2))}{(2n+3)!} \\ &= \frac{6^n(-4n^2 + 10n + 12)}{(2n+3)!} \\ &= \frac{-6^n(4n^2 - 10n - 12)}{(2n+3)!} \quad ? \end{aligned}$$

[6]

$$\left(\frac{x-6}{2x-7}\right)' = \frac{1(2x-7) - (x-6)2}{(2x-7)^2} = \frac{5}{(2x-7)^2} > 0$$

$f(x) = \frac{x-6}{2x-7}$ IS INCREASING

so $\left\{\frac{n-6}{2n-7}\right\}$ IS INCREASING + MONOTONIC

[3][a]

$$\frac{2n^2 + n^{\frac{3}{7}}}{3n^2 + n^{\frac{7}{3}}} > 0 \quad \left(\frac{1}{2}\right)$$

$$\frac{2n^2}{n^{\frac{7}{3}}} = \frac{2}{n^{\frac{1}{3}}}$$

$$\lim_{n \rightarrow \infty} \frac{2n^2 + n^{\frac{3}{7}}}{3n^2 + n^{\frac{7}{3}}} \cdot \frac{n^{\frac{7}{3}}}{2n^2} = \lim_{n \rightarrow \infty} \frac{2 + n^{-\frac{4}{7}}}{2} \cdot \frac{1}{3n^{-\frac{1}{3}} + 1} = 1 \quad \left(\frac{1}{2}\right)$$

$$2 \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{3}}} \text{ DIV (p-SERIES)} \quad (1)$$

so

$$\sum_{n=1}^{\infty} \frac{2n^2 + \sqrt[7]{n^3}}{3n^2 + \sqrt[3]{n^7}} \text{ DIV (LIMIT COMPARISON)} \quad (1)$$

$$[b] \lim_{n \rightarrow \infty} (2 - e^{\frac{1}{n}})^n = \boxed{e^{\lim_{n \rightarrow \infty} n \ln(2 - e^{\frac{1}{n}})}} = \boxed{e^{-1} \neq 0}$$

①
①
②

so $\sum_{n=1}^{\infty} (2 - e^{\frac{1}{n}})^n$ DIV (DIVERGENCE)

②
①

$$\lim_{x \rightarrow \infty} x \ln(2 - e^{\frac{1}{x}}) = \boxed{\lim_{x \rightarrow \infty} \frac{\ln(2 - e^{\frac{1}{x}})}{\frac{1}{x}}} = \boxed{\lim_{x \rightarrow \infty} \frac{-e^{\frac{1}{x}}}{2 - e^{\frac{1}{x}}}}$$

①
①

$$= \boxed{-1}$$

②

$$[c] \quad \boxed{-1 \leq \cos n \leq 1} \quad \textcircled{\frac{1}{2}}$$

$$\boxed{3 \leq 7 - 4 \cos n \leq 11} \quad \textcircled{1}$$

$$\textcircled{1} \quad \boxed{(7 - 4 \cos n) e^{-n} \leq 11 e^{-n}} = \frac{11}{e^n}$$

$$\textcircled{\frac{1}{2}} \quad \boxed{\sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n \text{ CONV}} \quad \left(\text{GEOMETRIC} \quad \left| r \right| = \frac{1}{e} \right)$$

$\textcircled{1} \qquad \qquad \qquad \textcircled{\frac{1}{2}}$

$$\text{SO} \quad \boxed{\sum_{n=1}^{\infty} (7 - 4 \cos n) e^{-n} \text{ CONV}} \quad \left(\text{COMPARISON} \right)$$

$\textcircled{\frac{1}{2}} \qquad \qquad \qquad \textcircled{1}$